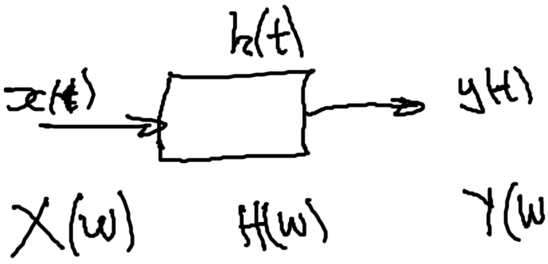
Sistemas Lineares LIT

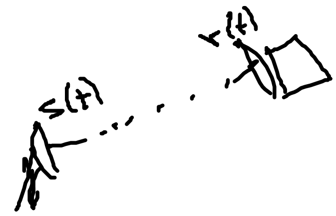


$y(t) = x(t) * h(t)$

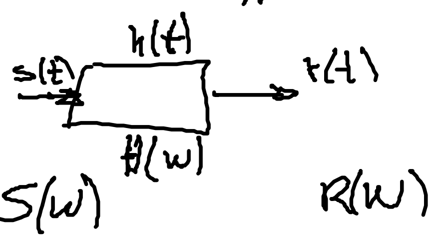
$Y(w) = X(w) . H(w)$

s(t)

$$\boxed{r(t) = \alpha\, s(t - t_0)}$$



$R(w) = H(w) . S(w)$



$$R(w) = \alpha\, S(w) . e^{-jwt_0} \Rightarrow H(w) = \frac{R(w)}{S(w)} = \alpha\, e^{-jwt_0}$$

$$h(t) = \mathcal{F}^{-1}\{H(w)\}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} H(w)\, e^{jwt}\, dw$$

$$= \frac{1}{2\pi} \int_{\infty}^{\infty} \alpha\, e^{-jwt_0} . e^{jwt}\, dw$$

$$= \frac{\alpha}{2\pi} \int_{-\infty}^{\infty} e^{jw(t - t_0)}\, dw$$

$$= \frac{\alpha}{2\pi} \left[\frac{1}{j(t - t_0)} e^{jw(t - t_0)} \Big|_{-\infty}^{\infty} \right]$$

$$\boxed{\int e^{ax}\, dx = \frac{1}{a} e^{ax}}$$

$$h(t) = \frac{\alpha}{j2\pi(t-t_0)}\left[\delta(t-t_0) - 0\right] =$$

$$\boxed{h(t) = \frac{\alpha}{j2\pi(t-t_0)}\,\delta(t-t_0)}$$

Ex.

$$y''(t) = 2y'(t) - x(t) \quad \leftarrow \text{SLIT}$$

$$X(\omega) \to \boxed{H(\omega)} \to Y(\omega)$$

$$Y(\omega) = H(\omega).X(\omega)$$

$$j = \sqrt{-1}$$

$$j^2 = -1$$

$$\frac{d^2y(t)}{dt^2} = 2.\frac{dy(t)}{dt} - x(t)$$

$$(j\omega)^2 Y(\omega) = 2.j\omega Y(\omega) - X(\omega)$$

$$-\omega^2.Y(\omega) - j2\omega Y(\omega) = -X(\omega)$$

$$(\omega^2 + j2\omega)\,Y(\omega) = X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{\omega^2 + j2\omega}$$

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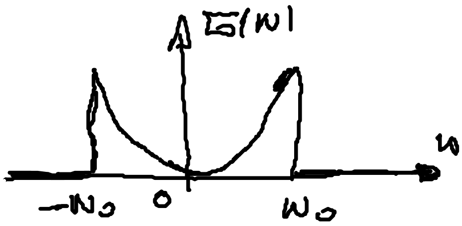
9)

$$G(\omega) = \begin{cases} \omega^2, & |\omega| \le \omega_0 \quad \leftarrow \quad -\omega_0 \le \omega \le \omega_0 \\ 0, & |\omega| > \omega_0 \quad \leftarrow \quad -\omega_0 > \omega > \omega_0 \end{cases}$$



$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\,e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} \omega^2.\,e^{j\omega t}\,d\omega$$

$$** = \int \omega^2 e^{j\omega t}\,d\omega =$$

$$= \frac{1}{jt}\,\omega^2 e^{j\omega t} - \frac{2}{jt}\int \omega\, e^{j\omega t}\,d\omega$$

$$= \frac{1}{jt}\,\omega^2 e^{j\omega t} - \frac{2}{jt}\left[\frac{1}{jt}\,\omega\, e^{j\omega t} - \frac{2}{jt}\int e^{j\omega t}\,d\omega\right.$$

$$= \frac{1}{jt} w^2 e^{jwt} + \frac{2}{t^2} w e^{jwt} - \frac{4}{t^2} \frac{1}{jt} e^{jwt}$$

$$** = \left[\frac{1}{jt} w^2 + \frac{2}{t^2} w^2 - \frac{4}{jt^3} \right] e^{jwt}$$

$$g(t) = \frac{1}{2\pi} \left[\left(\frac{1}{jt} w_o^2 + \frac{2}{t^2} w_o^2 - \frac{4}{jt^3} \right) e^{jw_o t} - \left(\frac{1}{jt} w_o^2 + \frac{2}{t^2} w_o^2 - \frac{4}{jt^3} \right) e^{-jw_o t} \right]$$

$$g(t) = \frac{1}{2\pi} \left[\left(\frac{1}{jt} w_o^2 + \frac{2}{t^2} w_o^2 - \frac{4}{jt^3} \right) \left(e^{jw_o t} - e^{-jw_o t} \right) \right]$$